

**Your Written Report on this Project is Due
at 4:00 pm Friday December 20, 2013.**

This document will outline my expectations and guidelines for the applied probability project. You will work in pairs on this project, and each pair will submit one final report. Your final report will be graded according to the weights 70% for mathematical content and 30% on presentation.

The applied probability project is a mathematical endeavor, but it is also a technical writing assignment. Your formal report should be of professional quality — typed, having a cover sheet, introduction, body, and results and conclusions. Your paper should be well-written with good use of both technical and non-technical language, and displaying appropriate graphics where applicable.

Some Background to the Problem

A casino has been in operation for a few years and the number of gamblers visiting its tables, bars, and restaurants is lagging. So the owners of the casino are looking into ways to attract new gamblers. One bit of research has shown that some would-be gamblers are put off by the complexity of the existing games. Therefore, the casino owners are suggesting and wanting to explore a very simple new game that they have tentatively named “Suit or Rank”. They want to hire YOU, with your knowledge of probability, to go through the game thoroughly, analyzing and answering many probabilistic questions. The game is very simple and even coarse, but that is the point— hopefully this simplicity will attract the timid gambler.

The Basics of the Game

The game “Suit or Rank” (hereafter called “the game” for brevity) is a card game that starts with two common decks of 52 cards; both decks are well-shuffled and one deck is the gambler’s deck (the gambler will denote the visitor to the casino), the other deck is the dealer’s. Naturally the dealer works for the casino, and the game will be one of gambler versus dealer.

The game starts with the dealer dealing out two cards from the gambler’s deck and giving them to the gambler, and the gambler looks at these cards. Based on these two cards, the gambler chooses to either play “Suit” or play “Rank”, which are described below.

If the gambler decides to play Suit, then ranks don’t matter at all for the rest of the hand. After declaring his desire to play Suit, the gambler is dealt a third card from his deck. All that matters are the suits of these three cards. So, for example, if the gambler has the hand $\{7\spadesuit, 10\heartsuit, J\clubsuit\}$, then the gambler’s hand is actually viewed as $\{\spadesuit, \heartsuit, \clubsuit\}$. Now the dealer deals three cards at random from the dealer’s deck and then the gambler and the dealer compare the suit sets of their hands to determine a winner of the hand. In terms of value,

$$\spadesuit > \heartsuit > \diamondsuit > \clubsuit,$$

borrowing from bridge. Furthermore, multiplicity of a suit beats singularity in a suit. Therefore, here are several examples of hand comparisons, and the winner and loser:

$$\text{winner : } \{\spadesuit, \spadesuit, \spadesuit\} \quad \text{loser : } \{\heartsuit, \heartsuit, \heartsuit\}$$

$$\text{winner : } \{\spadesuit, \spadesuit, \diamondsuit\} \quad \text{loser : } \{\spadesuit, \spadesuit, \clubsuit\}$$

$$\text{winner : } \{\clubsuit, \clubsuit, \clubsuit\} \quad \text{loser : } \{\spadesuit, \spadesuit, \heartsuit\}$$

$$\text{winner : } \{\spadesuit, \heartsuit, \diamondsuit\} \quad \text{loser : } \{\heartsuit, \diamondsuit, \clubsuit\}$$

$$\text{winner : } \{\diamondsuit, \diamondsuit, \heartsuit\} \quad \text{loser : } \{\spadesuit, \diamondsuit, \heartsuit\}$$

It is important to note that the dealer will always win if there is a tie. This exhausts all the particulars if the gambler decides to play Suit.

After looking at his two initial cards, the gambler may choose to play “Rank”. If the gambler decides to play Rank, then suits don’t matter at all for the rest of the hand, and all attention is placed on the ranks. In terms of ranks, aces are the highest rank possible, then king, queen, jack, 10, 9, ... , 3, 2. After choosing to play Rank, the gambler gets a third card from his deck. Finally, the dealer deals himself three cards from the dealer’s deck and they compare. Here are several possible results:

winner : $\{7, 7, 7\}$ loser : $\{A, A, K\}$

winner : $\{A, A, 10\}$ loser : $\{A, A, 9\}$

winner : $\{4, 4, 2\}$ loser : $\{A, K, Q\}$

winner : $\{K, 3, 2\}$ loser : $\{Q, J, 5\}$

winner : $\{8, 6, 3\}$ loser : $\{8, 6, 2\}$

Again, the dealer wins in the event of a tie.

This game is simple to understand, but as you can probably guess, the casino needs to know *everything that can happen* and the *probability of all that can happen* way before it would ever introduce such a game. Pun intended, nothing can be left to chance.

Your assignment for this project is to completely analyze this game. The more details you can provide the better, but at a minimum the casino wants you to solve and present a thorough analysis of the following questions:

1. Is it ever advantageous at all for the gambler to choose to play Suit? If not then the game is not interesting and would be an embarrassment to the casino to introduce. You must rigorously justify your answer.
2. Find the probability that the gambler wins the hand if he is initially dealt two aces. Repeat your calculations for the case when the gambler is initially dealt two kings; two queens; two jacks; ... ; two 3s; two 2s. You should assume the gambler always chooses optimally from playing either Suit or Rank, and you should justify optimality. Make a table and graph of your findings.
3. Find the probability that the gambler wins the hand if he is initially dealt two ♠. Repeat your calculations for the case when the gambler is initially dealt two ♥; two ♦; two ♣. You should assume the gambler always chooses optimally from playing either Suit or Rank, and you should justify optimality. Make a table and graph of your findings.
4. Questions 2 and 3 above look at the extreme cases on the rank and suit scale. What about mediocre hands in the middle of the spectrum? Toward this end, can you make a recommendation to the gambler as to which choice, to play Suit or Rank, he should make, if his initial two cards are:

— $\{A♠, 7♥\}$

— $\{3♣, 2♥\}$

— $\{J♦, 10♠\}$

5. Can you generalize your results in question 4 as to the optimal choice for the gambler, to play Suit or Rank, when his initial two cards are $\{Ys, Xt\}$, where $Y > X$ are ranks and $s \neq t$ are suits.